STUDENT'S NAME: ____________________________

TEACHER'S NAME: ____________________________

2022

HURLSTONE AGRICULTURAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE ASSESSMENT TASK 4

Mathematics Advanced

General Instructions	• Reading time – 10 minutes • Working time – 3 hours • Write using a black or blue pen • NESA approved calculators may be used • A reference sheet is provided in the Section I booklet • For questions in Section II, show all relevant mathematical reasoning and/or calculations • This examination paper is not to be removed from the examination centre
Total marks: 100	**Section I – 10 marks** (pages 3 – 7) • Attempt Questions 1 – 10. The multiple-choice answer sheet has been provided • Allow about 15 minutes for this section **Section II – 90 marks** (pages 13– 44) • Attempt Questions 11 – 16, write your solutions in the spaces provided. • There are 6 separate question/answer booklets. Extra working pages are available if required. • Allow about 2 hours and 45 minutes for this section.

Disclaimer: Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2022 HSC Mathematics Advanced Examination.

SECTION I **(10 marks)**

Attempt Questions 1 - 10

Allow about 15 minutes on this section.

Use the multiple-choice answer sheet provided for Questions 1 – 10.

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1. Given $f(x) = x^2 + 4x^3$. For what values of x is the function decreasing?

(A) $-\frac{1}{6} < x < 0$

(B) $\frac{1}{6} > x > 0$

(C) $x > -\frac{1}{6}$ *or* $x < 0$

(D) $x < -\frac{1}{6}$ *or* $x > 0$

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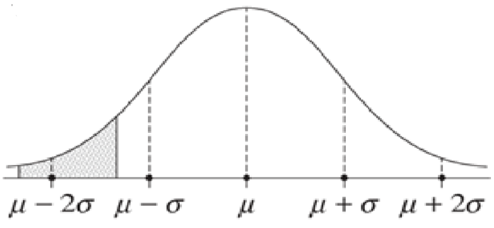
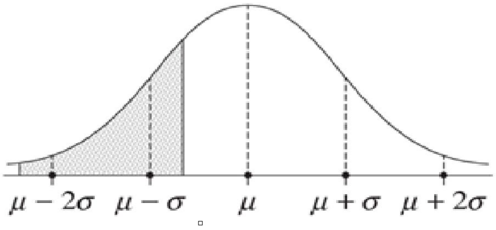
2. Suppose the weight of melons is normally distributed with a mean of μ and a standard deviation of σ.
A melon has a weight below the lower quartile of the distribution but NOT in the bottom 10% of the distribution.

Which of the following most accurately represents the region in which the weight of this melon lies?

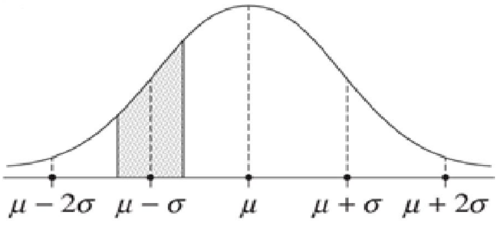
(A)



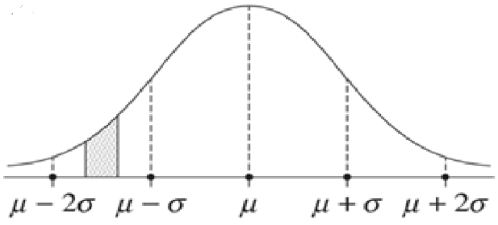
(B)



(C)



(D)



3. What is the difference between a discrete random variable and a continuous random variable?

(A) A discrete random variable takes all values in an interval of numbers while a continuous random variable has a fixed set of possible values with gaps between them.

(B) A discrete random variable has a fixed set of possible values with gaps between them while a continuous random variable takes all values in an interval of numbers.

(C) A discrete random variable takes only negative numbers while a continuous random variable takes both positive and negative numbers.

(D) A discrete random variable takes both positive and negative numbers while a continuous random takes only negative numbers.

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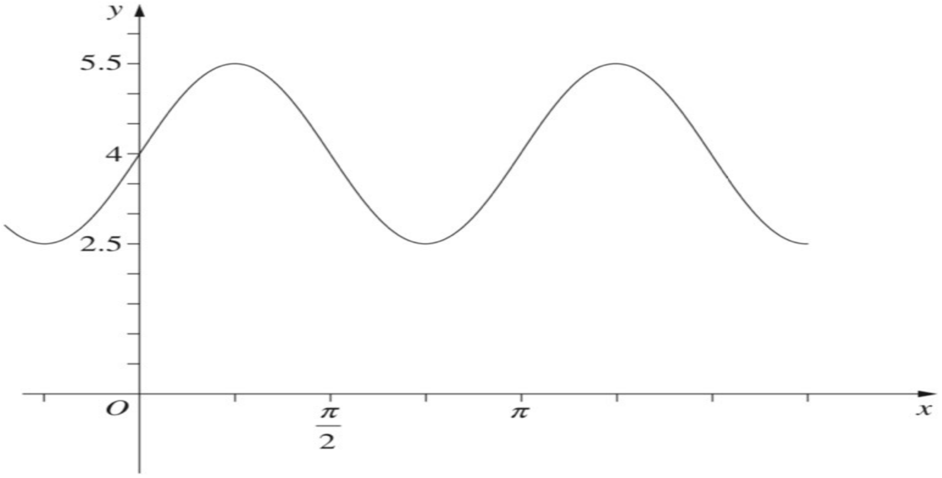
4. The diagram below shows part of the graph of $y = a\sin\left(bx\right) + 4$.



What are the values of *a* and *b*?

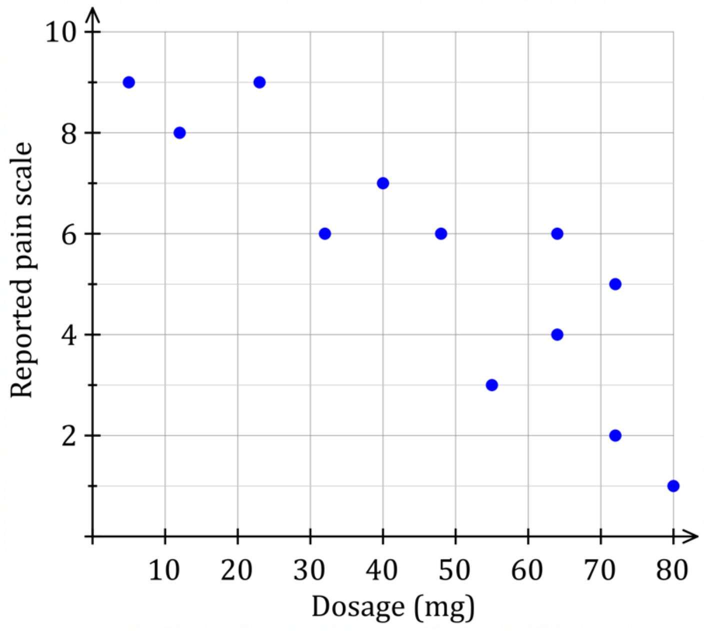
(A) $a = 3 \quad b = \frac{1}{2}$

(B) $a = 1.5 \quad b = \frac{1}{2}$

(C) $a = 3 \quad b = 2$

(D) $a = 1.5 \quad b = 2$

5. A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



How could you describe the correlation between the pain and the dosage?

(A) A moderate negative correlation

(B) A moderate positive correlation

(C) A weak positive correlation.

(D) No correlation.

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6. Given that $\int_2^6 f(x)dx = 3$, what is the value of $\int_4^6 f(2(x-3))dx$?

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) 3

(D) 8

7. What is the value of the derivative of $y = 2\sin 3x - 3\tan x$ at $x = 0$?

(A) -1

(B) 0

(C) 3

(D) -9

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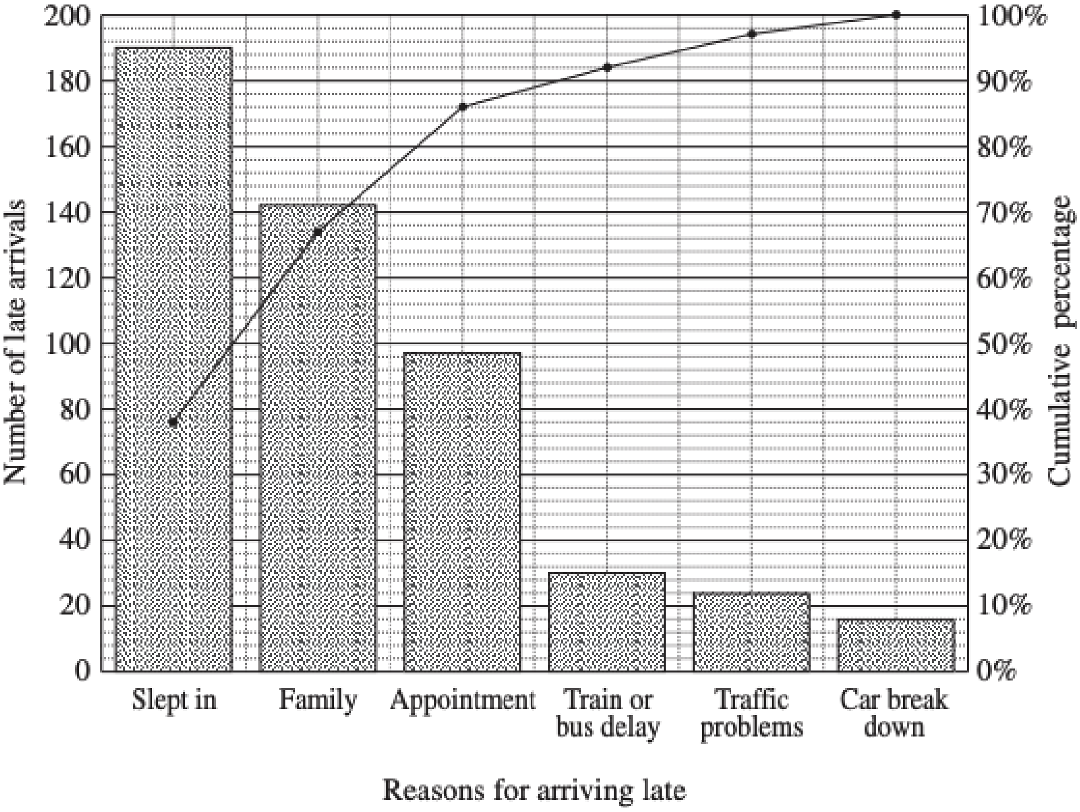
8. A school collected data related to the reasons given by students for arriving late. The Pareto chart shows the data collected.



What percentage of students gave the reason 'Appointment'?

(A) 24%

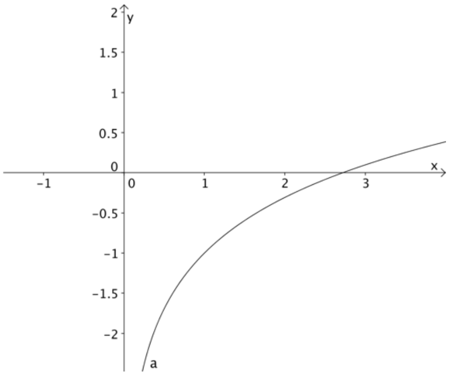
(B) 19%

(C) 42%

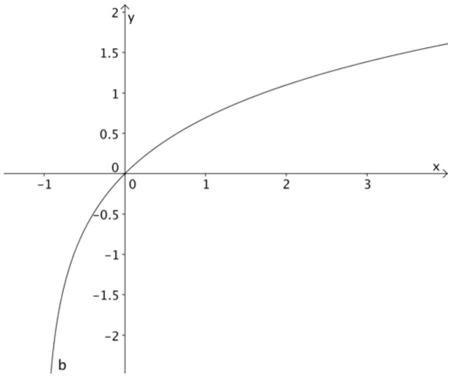
(D) 86%

9. The graph of the function $y = \ln(x + 1)$ is shown in which of the graphs below?

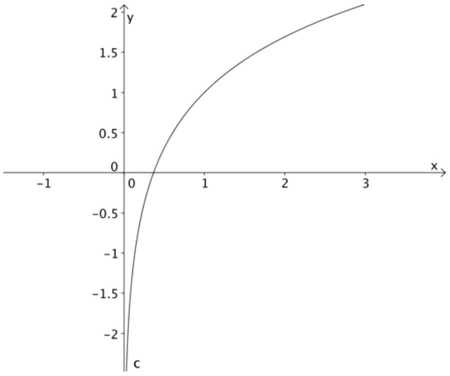
(A)



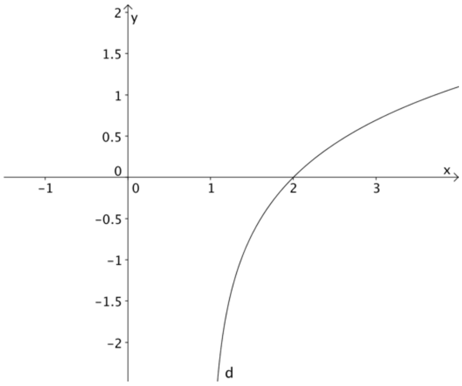
(B)



(C)



(D)



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10. What is the derivative of the function $y = \left(e^{2x+1}\right)^3$?

(A) $\left(2e^{2x+1}\right)^3$

(B) $3\left(e^{2x+1}\right)^2$

(C) $3\left(e^{2}\right)^2$

(D) $6\left(e^{6x+3}\right)$

SECTION II **Name:** ________________________________

90 marks

Attempt Questions 11 - 16

Allow about 2 hours and 45 minutes on this section.

Answer each question in the spaces provided. Extra working space is available after each question. If you need to use this extra space, you must clearly indicate this in the main solution space, and then clearly indicate the question number and part that you are answering in the extra space.

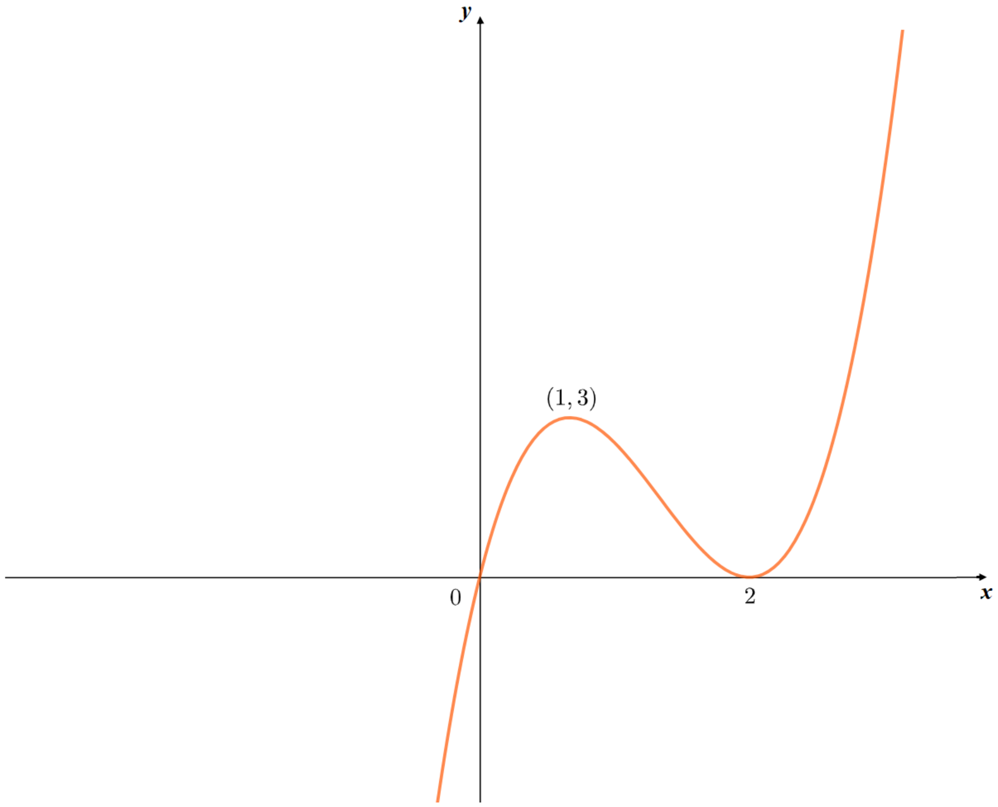
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

2022 Mathematics Advanced Trial Examination Section II

Question 11 (15 marks)

Marks

(a) The graph of $y = f(x)$ is shown below **2**



On the graph above sketch the graph of $y = 2f(-x)+1$, labelling the y-intercept and the co-ordinates of the stationary points.

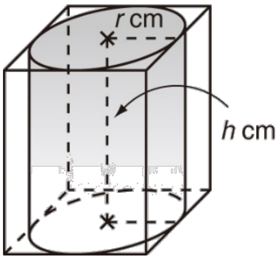
(b) Consider the function $f(x)=(x-2)^2(x+1)$

(i) Find the two stationary points and determine their nature. **3**

(ii) Sketch the graph of the function, clearly showing the stationary points and the x and y intercepts. **2**

(c) In the figure shown below, a cylinder of radius r cm and height h cm is inscribed in a closed container in the shape of a rectangular prism with a square base.

The total surface area of the container is 216 cm^2.



(i) Show that $h = \dfrac{27 - r^2}{r}$. **2**

(ii) Show that the volume of the cylinder is given by **1**

$$V = 27\pi r - \pi r^3$$

(iii) Hence, find the value of r such that the volume of the cylinder is greatest. **3**

(d) The equation of a curve C is given by $y = x^2 + ax + b$, where a and b are constants. **2**

The tangent to C at (0, 5) passes through (5, 20). Find the values of a and b.

End of Question 11

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: ______________________________

Question 12 (15 marks) **Marks**

(a) The tenth term of an arithmetic series is 29 and the fifteenth term is 44

(i) What is the common difference? **1**

(ii) What is the first term? **1**

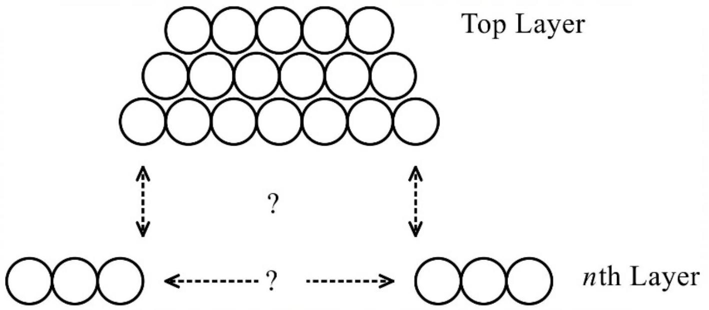
(iii) Find the sum of the first 85 terms. **1**

(b) For the given series $(2^1+3)+(2^2+3)+(2^3+3)+(2^4+3)+\ldots\ldots$. **2**

Show that the sum of the first n terms is given by $2^{n+1}-2+3n$

(c) Jayden works in a Moolworths store and he is making a stack of oranges against a sloping display panel.

The oranges are stacked in layers, as shown, where each layer contains one orange less than the layer below it.



When he has finished, there are five oranges in the top layer, six in the next and so on.
There are n layers altogether.

(i) Show there are $\frac{1}{2}n(n+9)$ oranges in the stack. **2**

(ii) If Jayden has 300 oranges to create his display, how many full rows can he create, if the top row still contains five oranges? **2**

(d) Consider the geometric series $1+\left(\sqrt{7}-2\right)+\left(\sqrt{7}-2\right)^2+......$

(i) Show with calculations why this geometric series has a limiting sum. **1**

(ii) Find the exact value of the limiting sum with a rational denominator. **2**

(e) Evaluate the definite integral $\int_0^2 2x^2\left(x^3-8\right)^3\,dx$. Give your answer as a fraction in its simplest form. **3**

End of Question 12

Name: ______________________________

Question 13 (15 marks) **Marks**

(a) X is a continuous random variable that is defined by the probability density function

$$f(x)=\begin{cases} k(x-1)^2 & 2\le x\le 4 \\ 0 & \text{otherwise} \end{cases}$$

(i) Show that $k=\dfrac{3}{26}$. **2**

(ii) Find the cumulative distribution function of X. **2**

(iii) Show that the upper quartile is approximately 3.748. **2**

(b) In Broken Hill, the maximum temperature for each day has been recorded. The mean of these maximum temperatures during spring is 25.8°C, and their standard deviation is 4.2°C.

(i) What temperature has a z-score of –1?. **1**

(ii) What percentage of spring days in Broken Hill would have maximum temperatures between 21.6°C and 38.4°C? **1**

(c) A continuous random variable X has a uniform distribution over the interval $(4,7)$.

Find $P(4.1 \leq X \leq 4.8)$ 1

(d) The weight, in grams, of beans in a tin is normally distributed with a mean μ and standard deviation of 7.8.

Given that 10% of tins contain less than 200 g, find the value of μ 2

Use the z score table extract given below.

z	**0.09**	**0.08**	**0.07**	**0.06**	**0.05**	**0.04**	**0.03**	**0.02**	**0.01**	**0.00**
–2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
–2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
–2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
–1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
–1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
–1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
–1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
–1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
–1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
–1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
–1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
–1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
–1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
–0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
–0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119

(e) The lifetime X (in years) of an electric light bulb has a probability distribution given by

$$f(x)=\begin{cases}\dfrac{6}{125}x(5-x) & 0\le x\le 5\\ 0 & \text{otherwise}\end{cases}$$

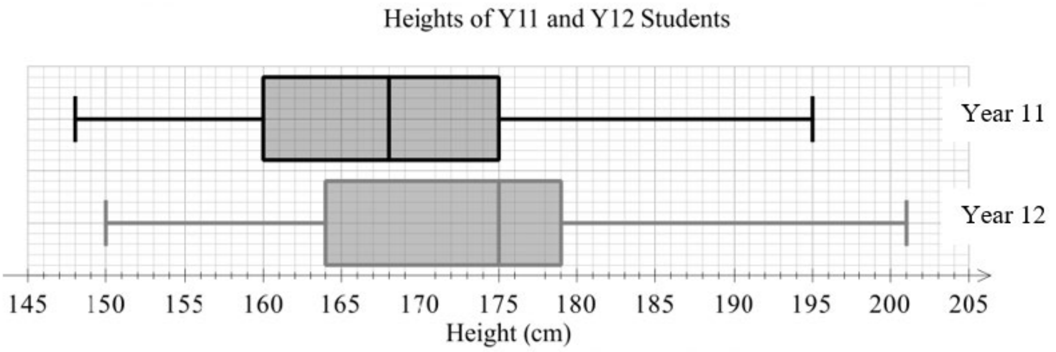
(i) Find the probability that the light bulb fails in within two year? **2**

(ii) Given that a standard lamp is fitted with two such new bulbs and that their failures are independent, find the probability that exactly one bulb fails within two years? **2**

End of Question 13

Question 14 (15 marks) **Marks**

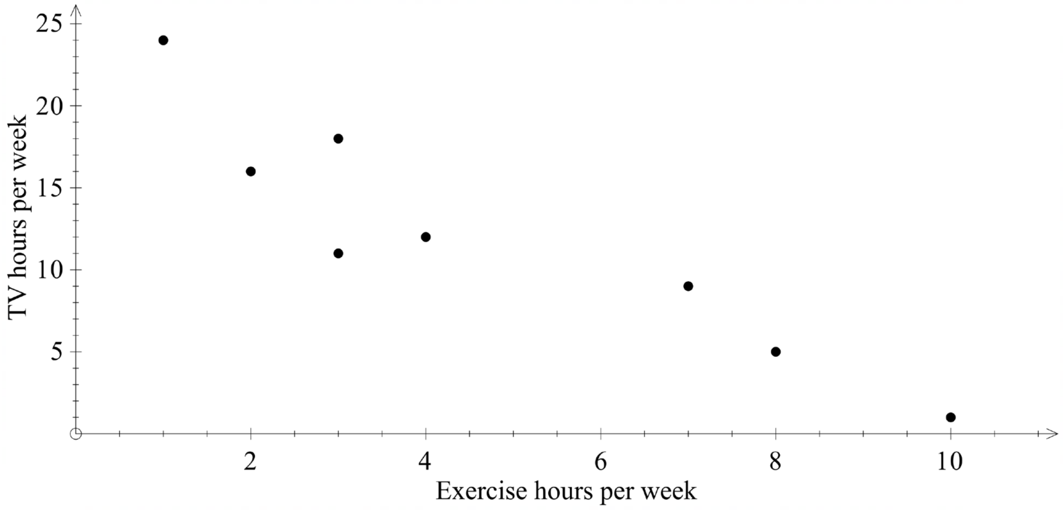
(a) The box-plot below shows the heights of students in Year 11 and Year 12 at a particular school.



There are the same number of students above 175 cm in Year 11 as there are students above 175 cm in Year 12. If there are 140 students in total in Year 11, how many students are in Year 12? **2**

(b) A group of children were surveyed on the number of hours each week that they spent exercising and watching television. The results are shown in the table and scatterplot below.

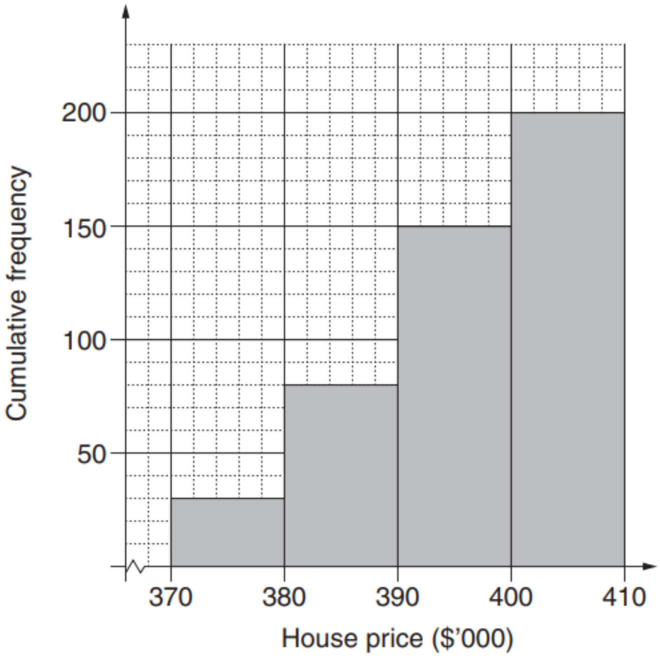
Exercise hours (*E*)	4	1	8	7	10	3	3	2
TV hours (*T*)	12	24	5	9	1	18	11	16



(i) Determine the equation of the least-squares regression line for this data using your calculator. Give your answers to two decimal places **2**

(ii) Using your equation from part (i), predict how many hours a week a child would spend exercising if they spend 15 hours watching television. **2**

(c) Data from 200 recent house sales are grouped into class intervals and a cumulative frequency histogram is drawn.



By completing the table, calculate the mean house price. **2**

Class Centre ($'000)	*Frequency*

(d) Consider the data set given in the frequency table below.

Score	Frequency
2	3
3	4
4	2
5	1

(i) Find the mean and population standard deviation for this data set. **2**

(ii) If a score of 9 is added to this data set, explain what will happen to the mean and standard deviation. Justify your answer. **2**

(e) The variables profit made and amount spent on advertising are strongly correlated with a correlation coefficient of $r = 0.9$. What conclusions can you draw from this information? **1**

(f) It is given that a data value below $Q_1 - 1.5 \times IQR$ or above $Q_3 + 1.5 \times IQR$ is the criteria for recognising the outliers. **2**
A set of data has a lower quartile (Q_1) of 10 and an upper quartile (Q_3) of 16.
What is the maximum possible range for this set of data if there are no outliers?

End of Question 14

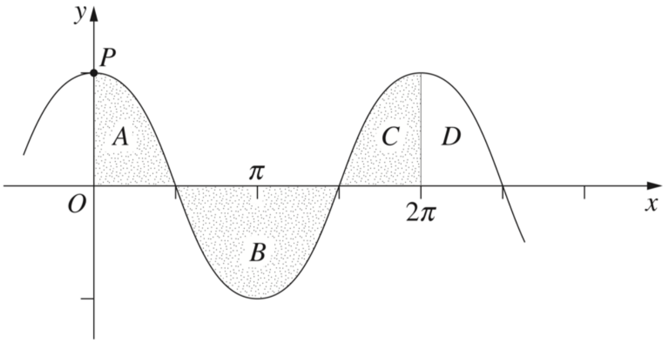
Name: ______________________________

Question 15 (15 marks) **Marks**

(a) If $\cos\alpha = -\dfrac{4}{5}$ and $\sin\alpha < 0$, find the exact value of $\tan\alpha$. **2**

(b) Solve $2\sin x\cos x = \sin x$ for $0 \le x \le 2\pi$. **3**

(c) The diagram below shows the graph $y = 2\cos x$.



(i) State the coordinates of P. **1**

(ii) Evaluate the integral $\int_{\frac{3\pi}{2}}^{2\pi} 2\cos x \, dx$. **2**

(iii) Indicate which area in the diagram, *A, B, C* or *D,* is represented by the integral $\int_{\frac{3\pi}{2}}^{2\pi} 2\cos x \, dx$. **1**

(iv) Using parts (ii) and (iii), or otherwise, find the area of the region bounded by the curve $y = 2\cos x$ and the x-axis, between $x = 0$ and $x = 2\pi$. **1**

(v) Using the parts above, write down the value of $\int_{\frac{\pi}{2}}^{2\pi} 2\cos x \, dx$. **1**

(d) Show that $\dfrac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta} = \sin\theta\tan\theta$ **2**

(e) Differentiate $\dfrac{\sin x}{x}$. **2**

End of Question 15

Question 16 (15 marks) **Marks**

(a) By first writing the function $y = 7^x$ in logarithmic form, show that its derivative is $7^x \ln 7$. **2**

(b) A particle is accelerating according to the function $a = -\dfrac{3}{t^2}$ ms^{-2}. **3**

If, when $t = 0.5$ s, its velocity, $v = 6$ ms^{-1} and its displacement, $x = 1$ m, find the particle's displacement function in simplest form.

(c)

(i) Solve the equation $e^{x-1}-2=0$, giving your answer correct to 2 decimal places. **1**

(ii) Evaluate the definite integral $\int_1^2 \left(e^{x-1}-2\right)dx$. **2**

(iii) Explain why your answer in (ii) does not give the area under the curve $y=e^{x-1}-2$ between the lines $x=1$ and $x=2$.
(Note: it is not necessary to calculate the area). **1**

(d) Find the gradient of the curve $y = xe^x$ at the point where $x = \ln 2$. **2**

(e) Consider the function defined by $f(x) = e^{2x}(1-x)$.

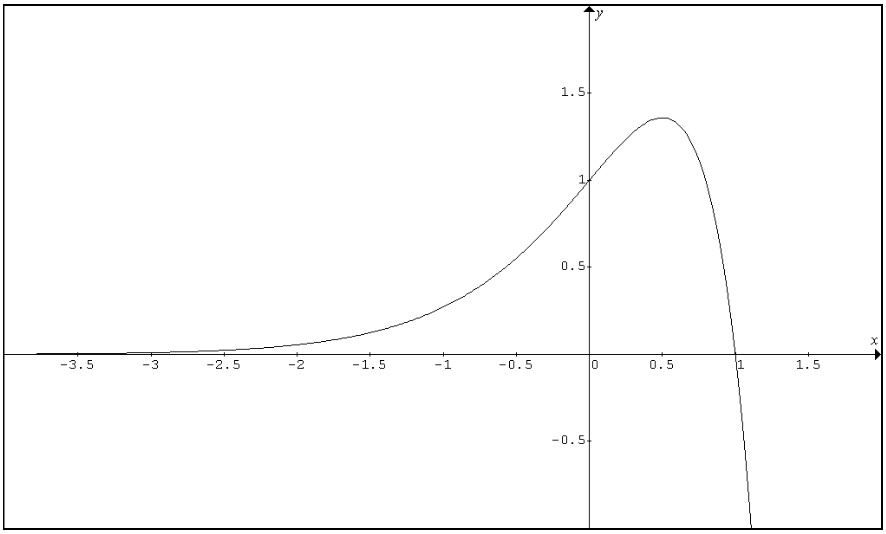
(i) Copy and complete the table of values, giving your answers correct to 2 dec. pl. **1**

x	–3	–2	–1	0	1
$f(x)$	0.01			1	0

(ii) Using the trapezoidal rule with five function values, approximate the area under the curve $y = f(x)$, between $x = -3$ and $x = 1$. **2**

(iii) The graph of $y = e^{2x}(1-x)$ is shown below.

1



Decide whether your approximation in (ii) above is an over-estimate or an under-estimate of the true value of the area under the curve.
Give a brief reason for your answer.
[Note: Do not attempt to integrate the function.]

End of Question 16

End of Paper

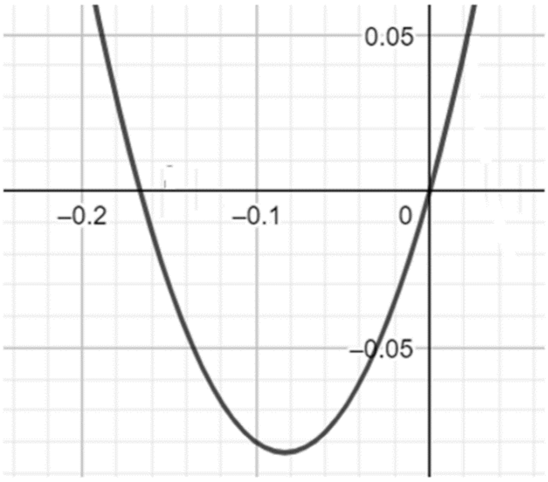
Yr 12 Mathematics Advanced Trial Marking Guidelines

Multiple Choice Solutions

Question 1

$f(x)=x^2+4x^3$

$f'(x)=2x+12x^2$



For the function decreasing, $f'(x)<0$,

then $2x+12x^2<0$

$2x(1+6x)<0$

$\therefore -\frac{1}{6}<x<0$

Answer *A*

Question 2

Answer *C*

Question 3

Answer *B*

Question 4

$y=a\sin\left(bx\right)+4$

The graph of $y=\sin x$ has been translated 4 units up.

Observing the given graph the maximum displacement of the graph from the mean position $\left(y=4\right)$ is ± 1.5 units.

$\therefore$ Amplitude $a=1.5$

Observing the given graph the period of the graph is π units

$Period=\frac{2\pi}{\pi}=2$

$\therefore b=2$

Answer *D*

Question 5

A moderate negative correlation

Answer *A*

Question 6

$\int_2^6 f(x)dx = 3$

As $f(2x)$ is a horizontal dilation of $f(x)$, the graph is compressed horizontally by a factor of $\frac{1}{2}$. This means that the range is unchanged but the domain is halved thereby halving the the area under the graph

$$\int_1^3 f(2x)dx = \frac{1}{2}\int_2^6 f(x)dx = \frac{3}{2}$$

Horizontally translating the graph three units to the right gives:

$$\int_4^6 f(2(x-3))\,dx = \frac{3}{2}$$

Answer *B*

Question 7

$$y = 2\sin 3x - 3\tan x$$
$$\frac{dy}{dx} = 2(3)\cos 3x - 3\sec^2 x$$
at $x = 0$
$$\frac{dy}{dx} = 6\cos 0 - 3\sec^2 0 = 6 - 3 = 3$$

Answer *C*

Question 8

Look at the change in cumulative percentage from ‘Family’ to ‘Appointment’.

Answer *B*

Question 9

Horizontally translate the graph of $y = \ln x$ one unit to the left.

When

$$x = 0$$
$$y = \ln(x+1) = \ln 1 = 0$$

The graph to pass through $(0,0)$

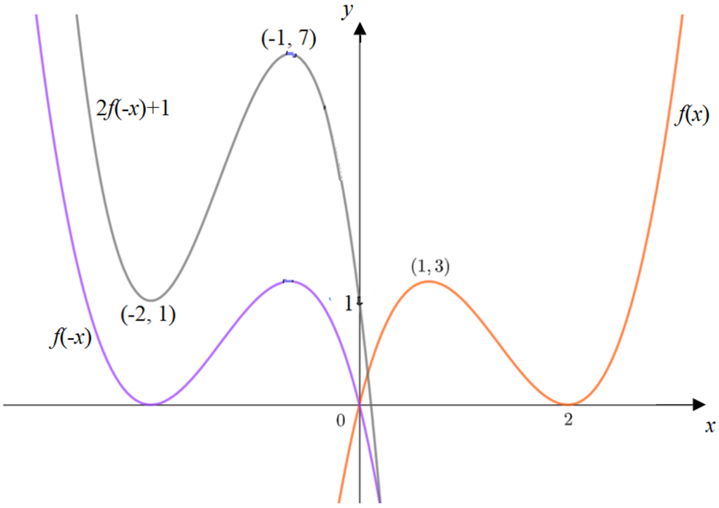
Answer *B*

Question 10

$$y = \left(e^{2x+1}\right)^3 = e^{6x+3}$$

$$\frac{dy}{dx} = 6e^{6x+3}$$

Answer *D*

2022 Y12 Trial Maths Advanced		
Question 11	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		
MA11-1 -uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems MA11-2 -uses the concepts of functions and relations to model, analyse and solve practical problems MA11-5 -interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems MA12-3 -applies calculus techniques to model and solve problems		
Outcome	**Solutions**	**Marking Guidelines**
MA11-1 MA11-2	a) 	2 marks for correct solution 1 mark for correct shape after transformation 1 mark for min, max and y-intercept
MA11-5	b)i) $f(x) = (x-2)^2(x+1)$ $f'(x) = (x-2)^2 + (x+1)\cdot 2(x-2)$ $= (x-2)(x-2+2x+2)$ $= (x-2)(3x)$ $f'' = (x-2)3 + (3x)$ $= 6x-6$ $= 3(x-2)$ Solving $f'(x) = 0$, we have $3(x-2) = 0$ $x = 2$ or 0 $at\ x = 2$ $f'' = 6(2) - 6$ $= 6$ > 0 $\Rightarrow$ Local min turning point at $(2, 0)$ $at\ x = 0$ $f'' = 6(0) - 6$ $= -6$ < 0 $\Rightarrow$ Local max turning point at $(0, 4)$	3 marks for correct solution 1 mark for correct first derivative 1 mark for finding max with explanation 1 mark for finding min with explanation

OR

Solving $f'(x) = 0$, we have $x = 2$ or 0

The signs of $f'(x)$ are summarized in the table below:

x	$x = -1$	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$f'(x)$	9	0	–3	0	9
	/	—	\	—	/

The sign of $f'(x)$ changes from positive to negative as x increases through 0.

$$f\left(0\right) = \left[\left(0\right) - 2\right]^2 \left[3\left(0\right)\right] = 4$$

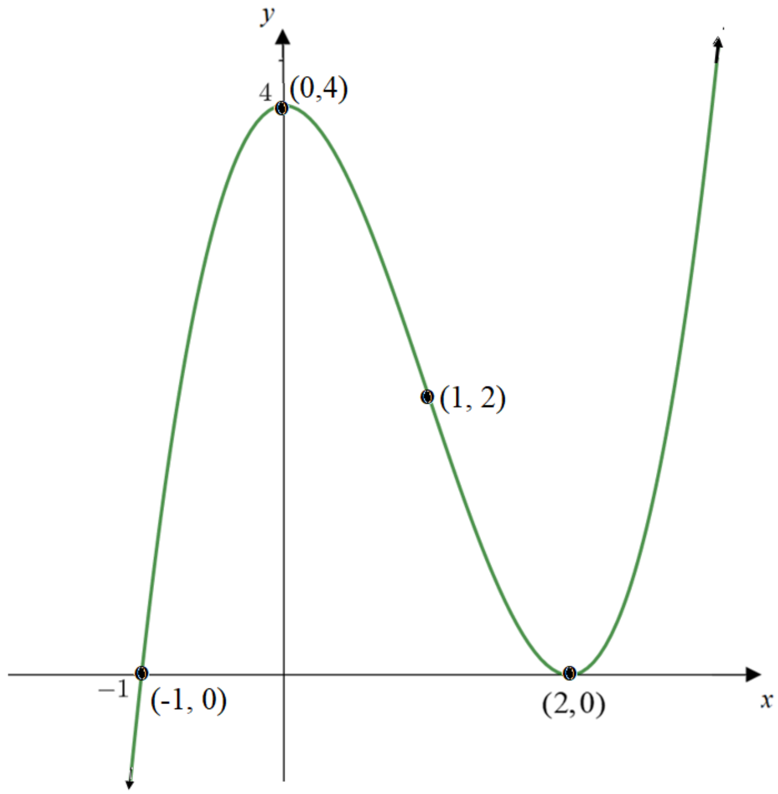
$\therefore$ Local maximum at (0, 4)

The sign of $f'(x)$ changes from negative to positive as x increases through 2.

$$f(2) = [(2) - 2]^2 [(2) + 3] = 0$$

$\therefore$ Local minimum at (2, 0)

(ii)



2 marks for correct solution
1 mark for the shape
1 mark for label y-intercept, max , min

MA12-3	c) i) Length of each side of the square base = diameter of the circular base = $2r$ Surface Area: $2(2r)^2+4(2rh)=216$ $h=\frac{27-r^2}{r}$	1 mark for showing h correctly
	ii) Let V cm^3 be the volume of the cylinder. $V=\pi r^2 h$ $=\pi r^2\cdot\frac{27-r^2}{r}$ $=27\pi r-\pi r^3$	1 mark for showing V correctly
	iii) $\frac{dV}{dr}=27\pi-3\pi r^2$ $\frac{d^2V}{dr^2}=-6\pi r$ Solving $\frac{dV}{dr}=0$, we have $r = 3$ or -3 $r = 3$ ($r > 0$) at $r = 3$ $\because \frac{d^2V}{dr^2}=-6\pi(3)<0$ ∴ V attains its maximum at $r = 3$. ∴ The required value of r is 3.	3 marks for correct solution 1 mark for correct first and second derivative 1 mark for finding r 1 mark for showing max volume of r
MA11-1	d) $y=x^2+ax+b$ $\frac{dy}{dx}=2x+a$ ∴ M_T at $x=0$, $\frac{dy}{dx}=\frac{20-5}{5-0}=3$ $2(0)+a=3$ $a=\underline{\underline{3}}$ Since (0, 5) lies on curve: $\therefore y=x^2+ax+b$ $5=0^2+a\times 0+b$ $\therefore b=5$	2 marks for correct solution 1 mark for correct derivative 1 mark for correct a

2022 Y12 Trial Maths Advanced

Question 12 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA12-4 -applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems
MA12-7 -applies the concepts and techniques of indefinite and definite integrals in the solution of problems

Outcome	**Solutions**	**Marking Guidelines**
MA12-4	**(a)(i)** Given an Arithmetic Series with $T_{10}=29$ and $T_{15}=44$, $a+9d=29 \quad [1]$ and $a+14d=44 \quad [2]$ Subtracting, $5d=15$ $\therefore$ common difference $d=3$	1 mark for correct answer
	(ii) Sub $d=3$ into $[1]$, $a+27=29$ $\therefore$ first term is 2	1 mark for correct answer
	(iii) Using $S_n=\frac{n}{2}\left(2a+(n-1)d\right)$, $S_{85}=\frac{85}{2}\left(2(2)+(85-1)3\right)=10880$	1 mark for correct answer
MA12-4	**(b)** Given $(2^1+3)+(2^2+3)+(2^3+3)+(2^4+3)+\ldots\ldots$ $=\left(2^1+2^2+2^3+2^4+\ldots\ldots..\right)+\left(3+3+3+3+\ldots\ldots.n \text{ times}\right)$ $\left(2^1+2^2+2^3+2^4+\ldots\ldots..\right)$ is a GP with $a=2, r=2$ $\therefore \text{Sum } S_n=\left(\frac{2\left(2^n-1\right)}{2-1}\right)=2\left(2^n-1\right)=2^{n+1}-2$ $\left(3+3+3+3+\ldots\ldots.n \text{ times}\right)$ is an Arithmetic Series with $a=3$ and $d=0$ $\therefore \text{Sum } S_n=3n$ $\therefore \text{Total Sum}=2^{n+1}-2+3n$	2 marks for correct solution 1 mark for substantial progress towards solution
MA12-4	**(c)(i)** Arithmetic Series $5+6+7+8+\ldots\ldots\ldots+n$ So $a=5 \quad d=6-5=1$ $S_n=\frac{n}{2}[a+l]$ $=\frac{n}{2}[5+(n+4)]$ $=\frac{n}{2}(n+9)$ $=\frac{1}{2}n(n+9)$	1 mark for establishing series and using correct formula 1 mark for correct answer

MA12-4	**(ii)** $S_n = \frac{1}{2}n(n+9)$ $300 = \frac{1}{2}n(n+9)$ $600 = n^2 + 9n$ $n^2 + 9n - 600 = 0$ $n = \frac{-9 \pm \sqrt{81 - 4 \times 1 \times -600}}{2}$ $n = 20.4 \text{ or } -29.4$ So Jayden can make 20 complete rows.	1 mark for setting up the correct quadratic equation 1 mark for correct answer
MA12-4	**(d)(i)** For geometric series to have a limiting sum $\left\lvert \frac{T_2}{T_1} \right\rvert < 1$ $\frac{\sqrt{7} - 2}{1} \approx 0.6$ $\therefore \quad \lvert r \rvert < 1$ $\therefore$ has a limiting sum	1 mark for showing |r|<1
MA12-4	**(ii)** $S = \frac{a}{1-r}$ $= \frac{1}{1-(\sqrt{7}-2)}$ $= \frac{1}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$ $= \frac{3+\sqrt{7}}{9-7}$ $= \frac{3+\sqrt{7}}{2}$	1 mark for correct formula and substitution 1 mark for correct rationalised answer
MA12-7	**(e)** $\int_0^2 2x^2\left(x^3-8\right)^3 dx$ $= \frac{2}{3}\int_0^2 3x^2\left(x^3-8\right)^3 dx \qquad \text{using } \int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1}$ $= \frac{2}{3}\left(\frac{\left(x^3-8\right)^4}{4}\right)_0^2$ $= \frac{2}{3}\left(\frac{\left(2^3-8\right)^4}{4}\right) - \frac{2}{3}\left(\frac{\left(0^3-8\right)^4}{4}\right)$ $= \frac{2}{3}(0) - \frac{2}{3}(1024)$ $= -\frac{2048}{3}$	3 marks for correct solution 2 marks for making substantial progress toward the solution 1 mark for making some progress towards the solution

Year 12 Mathematics Advanced Assessment Task 4 2022	
Question No. 13 Solutions and Marking Guidelines	
Outcomes Addressed in this Question	
MA12-8 solves problems using appropriate statistical processes	
Solutions	**Marking Guidelines**
(a) (i) $\int_2^4 k(x-1)^2 \mathrm{d}x = 1$ $k\int_2^4 (x-1)^2 \mathrm{d}x = 1$ $k\left[\frac{(x-1)^3}{3}\right]_2^4 = 1$ $k\left(9-\frac{1}{3}\right) = 1$ $k\left(\frac{26}{3}\right) = 1$ $k = \frac{3}{26}$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution.
(ii) $CDF = \int_2^x \frac{3}{26}(x-1)^2 dx$ $= \frac{3}{26}\left[\frac{(x-1)^3}{3}\right]_2^x$ $= \frac{3}{26}\left[\frac{(x-1)^3}{3} - \frac{1}{3}\right]$ $= \frac{1}{26}\left[(x-1)^3 - 1\right]$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution.
(iii) $\frac{(x-1)^3 - 1}{26} = 0.75$ $(x-1)^3 - 1 = 19.5$ $(x-1)^3 = 20.5$ $x - 1 = 2.736851\ldots$ $x = 3.736851$ $x \sim 3.739$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the correct solution.

b) (i) $-1 = \frac{x-25.8}{4.2}$ $x = 21.6$	**Award 1** mark for the correct solution.
(ii) From $z-$ score -1 to mean ($z-$ score 0) $= \frac{1}{2} \cdot 68\% = 34\%$ From mean ($z-$ score 0) to $z-$ score 3 $= \frac{1}{2} \cdot 99.7\% = 49.85\%$ Total $= 34\% + 47.85\%$ $= 83.85\%$	**Award 1** mark for the correct solution.
(c) $P(4.1 \leq X \leq 4.8) = \frac{4.8-4.1}{7-4}$ $= \frac{7}{30}$	**Award 1** mark for the correct solution.
(d) $-1.28 = \frac{200 - \mu}{7.8}$ $200 - \mu = -9.984$ $\mu = 209.984$ $\mu = 210g$ (nearest whole value)	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the correct solution.

(e) (i) $F(x)=\int_0^x \frac{6}{125}\left(5x-x^2\right)\,\mathrm{d}x$ $=\frac{6}{125}\left[\frac{5x^2}{2}-\frac{x^3}{3}\right]$ $P(X\leq 2)=F(2)$ $P(X\leq 2)=\frac{6}{125}\left(\frac{5}{2}\cdot(2)^2-\frac{(2)^3}{3}\right)$ $P(X\leq 2)=\frac{44}{125}$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the correct solution.
(ii) $\textit{Fails in two years}=\frac{44}{125}=P(B)$ $\textit{Does not fail}=1-\frac{44}{125}$ $=\frac{81}{125}=P(B')$ $P(BB')+P(B'B)=\frac{44}{125}\cdot\frac{81}{125}+\frac{81}{125}\cdot\frac{44}{125}$ $=\frac{7128}{15625}$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the correct solution.

Year 12	Mathematics Advanced	Assessment Task 4 2022 HSC
Question No. 14	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

Statistics and Bivariate Data Analysis MA-S2
(S2.1 Data and summary statistics; S2.2 Bivariate Data Analysis)

<table>
<tr><th>Part</th><th>Solutions</th><th>Marking Guidelines</th></tr>
<tr><td>(a)</td><td>No. of Year 11 students above 175cm $= 25\% \times 140$
$= 35$
as 50% of Year 12 students are above 175cm

$50\% \times$ No. Year 12 students $= 35$
No. Year 12 students $= 70$</td><td><u>2 marks</u>: correct solution.

<u>1 mark</u>: correctly identifies number of Year 11 students above 175cm or equivalent merit</td></tr>
<tr><td>(b) (i)</td><td>By calculator, to 2 decimal places
$A = 22.10$
$B = -2.13$

Least Squares Regression Line
$y = Bx + A$
$y = -2.13x + 22.10$

Note: y is the TV hours and x is the exercise hours</td><td><u>2 marks</u>: correct solution

<u>1 mark</u>: correct A or correct B or line in form $y = Ax + B$</td></tr>
<tr><td>(b)(ii)</td><td>$y = -2.13x + 22.10$
$15 = -2.13x + 22.10$
$15 - 22.10 = -2.13x$
$-7.1 = -2.13x$
$x = \frac{-7.1}{-2.13}$
$x = 3.3$

Approximately 3.3 hours</td><td><u>2 marks</u>: correct solution

<u>1 mark</u>: significant progress made towards solution</td></tr>
<tr><td>(c)</td><td>
<table>
<tr><th><i>Class centre</i> ($’000)</th><th><i>Frequency</i></th></tr>
<tr><td>375</td><td>30</td></tr>
<tr><td>385</td><td>50</td></tr>
<tr><td>395</td><td>70</td></tr>
<tr><td>405</td><td>50</td></tr>
</table>
$\overline{x} = \frac{(375 \times 30) + (385 \times 50) + (395 \times 70) + (405 \times 50)}{200}$
$= \$392$
$\therefore$ Mean House Price = $392 000</td><td><u>2 marks</u>: correct solution

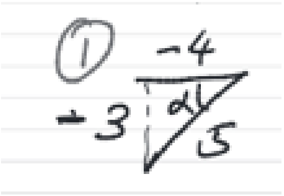
<u>1 mark</u>: significant progress towards correct solution</td></tr>
</table>

(d) (i)	From calculator: $\bar{x} = 3.1$ $\sigma = 0.9433981132$	**<u>2 marks</u>:** correct mean and standard deviation **<u>1 mark</u>:** either mean or standard deviation correct
(d)(ii)	Adding 9 to the set of scores and recalculating gives $\bar{x} = 3.\dot{6}\dot{3}$ $\sigma = 1.919882917$ so, both the mean and the standard deviation increase.	**<u>2 marks</u>:** correct statements for both mean and standard deviation with justification. **<u>1 mark</u>:** correct statements for both mean and standard deviation without justification or correct mean statement & justification or correct standard deviation statement & justification
(e)	Profit will increase as the money spent on advertising increases.	**<u>1 mark</u>:** correct statement
(f)	Interquartile range (IQR) = $Q_3 - Q_1$ $= 16 - 10$ $= 6$ Lower bound = $Q_1 - 1.5 \times IQR$ $= 10 - 1.5 \times 6$ $= 1$ Upper bound = $Q_3 + 1.5 \times IQR$ $= 16 + 1.5 \times$ $= 25$ ∴ Range = 25 – 1 = 24	**<u>2 marks</u>:** correct solution **<u>1 mark</u>:** significant progress towards correct solution

Year 12	Mathematics Advanced	Assessment Task 4 2022 HSC
Question No. 15	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

Trigonometric Functions MA-C3 C4 &T3
(T3 Trig functions and graphs; C3.1, C3.2, C4.1 and C4.2 Differentiation, Integration, and applications.)

Part	**Solutions**	**Marking Guidelines**
(a)	$\cos\alpha = -\frac{4}{5},\ \sin\alpha < 0$ $\therefore \alpha$ is in 4^{th} quadrant  $\tan\alpha = \frac{-3}{-4}$ ie $\tan\alpha = \frac{3}{4}$	**<u>2 marks</u>:** correct solution <u>with</u> justification **<u>1 mark</u>:** substantially correct solution
(b)	$2\sin x\cos x = \sin x$ $2\sin x\cos x - \sin x = 0$ $\sin x(2\cos x - 1) = 0$ $\sin x = 0 \qquad 2\cos x - 1 = 0$ $x = 0, \pi, \frac{5\pi}{3} \qquad \cos x = \frac{1}{2}$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$ so $x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$	**<u>3 marks</u>:** provides correct solution **<u>2 marks</u>:** correctly solves one case, or equivalent merit **<u>1 mark</u>:** identifies both cases to be considered or finds one correct answer, or equivalent merit.
(c)(i)	$A(0,2)$	**<u>1 mark</u>:** correct answer
(c)(ii)	$\int_{\frac{3\pi}{2}}^{2\pi} 2\cos x\,dx = \left[2\sin x\right]_{\frac{3\pi}{2}}^{2\pi}$ $= 2\sin 2\pi - 2\sin\frac{3\pi}{2}$ $= 0 - (-2)$ $= 2$	**<u>2 marks</u>:** correct solution **<u>1 mark</u>:** correct primitive or equivalent merit
(c)(iii)	**C**	**<u>1 mark</u>:** correct answer
(c)(iv)	$\text{Area} = 4 \times 2 \qquad (\text{area } A = \text{ area } C = 2 \times \text{area } B)$ $= 8 \text{ unit}^2$	**<u>1 mark</u>:** correct answer

(c)(v)	$\int_{\frac{\pi}{2}}^{2\pi} 2\cos x\,dx = 2-2\times 2 \qquad \left(\text{area A } - \text{ area B}\right)$ $= -2$	**1 mark:** correct answer
(d)	$\text{LHS} = \frac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta}$ $= \frac{\sec\theta\left(1-\cos^4\theta\right)}{1+\cos^2\theta}$ $= \frac{\sec\theta\left(1-\cos^2\theta\right)\left(1+\cos^2\theta\right)}{1+\cos^2\theta}$ $= \frac{1}{\cos\theta}\left(1-\cos^2\theta\right)$ $= \frac{1}{\cos\theta}\sin^2\theta$ $= \sin\theta\tan\theta$ $= \text{RHS}$	**2 marks:** correct solution **1 mark:** significant progress towards correct solution
(e)	$\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{u'v - v'u}{u^2}$ $= \frac{\left(\cos x\right)\times x - 1\times\sin x}{x^2}$ $= \frac{x\cos x - \sin x}{x^2}$	**2 marks:** correct solution. **1 mark:** attempts to use the quotient rule, or equivalent.

Year 12	Mathematics Advanced	Assessment Task 4 2022 HSC
Question No. 16	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

Exponential and Logarithmic Functions MA-C3 & C4
(C3.1, C3.2, C4.1 & C4.2 Differentiation, Integration and applications.)

Part	Solutions	Marking Guidelines
(a)	$y=7^x$ In logarithmic form $x=\log_7 y$ Changing the base $x=\frac{\ln y}{\ln 7}$ $\frac{dx}{dy}=\frac{1}{\ln 7}.\frac{1}{y}$ $=\frac{1}{y\ln 7}$ $\therefore \frac{dy}{dx}=y\ln 7 \quad \left(\text{since } \frac{dy}{dx}=\frac{1}{\left(\frac{dx}{dy}\right)}\right)$ $=7^x\ln 7$ as required	**2 marks:** correct solution **1 mark:** substantially correct solution
(b)	$a=-\frac{3}{t^2}$ $v=\frac{3}{t}+c$ but when $t=0.5,\ v=6$ ie. $6=\frac{3}{0.5}+c$ $6=6+c$ $c=0$ $\therefore v=\frac{3}{t}$ $x=3\ln t+c_2$ but when $t=0.5,\ x=1$ ie. $1=3\ln(0.5)+c_2$ $=3\ln\left(\frac{1}{2}\right)+c_2$ $=-3\ln 2+c_2$ $c_2=1+3\ln 2$ $\therefore x=3\ln t+3\ln 2+1$	**3 marks:** provides correct solution **2 marks:** makes significant progress towards solution **1 mark:** makes limited progress towards solution

<table>
<tr><td>(c)(i)</td><td>$e^{x-1}-2=0$
$e^{x-1}=2$
$\ln e^{x-1}=\ln 2$
$(x-1)\ln e=\ln 2$
$x-1=\ln 2$
$\therefore x=1+\ln 2\approx 1.69$</td><td><u>1 mark</u>: correct solution</td></tr>
<tr><td>(c)(ii)</td><td>$\int_1^2 e^{x-1}-2\,dx=\left[e^{x-1}-2x\right]_1^2$
$=\left(e^{2-1}-2\times 2\right)-\left(e^{1-1}-2\times 1\right)$
$=e-4-1+2$
$=e-3$</td><td><u>2 marks</u>: correct solution

<u>1 mark</u>: significant progress towards correct solution</td></tr>
<tr><td>(c)(iii)</td><td>The value of the integral, $e-3<0$ since between $x = 1$ and $x = 2$ (at $x=1+\ln 2$) the curve cuts the x axis. i.e., some of the area is below the x axis, the rest is above. The integral adds the “signed” areas. Actual area under the curve would need to be calculated by evaluating two integrals (between $x = 1$ and $x=1+\ln 2$ and between $x=1+\ln 2$ and x = 2) and adding their absolute values.</td><td><u>1 mark</u>: correct explanation</td></tr>
<tr><td>(d)</td><td>$y=xe^x$
$\frac{dy}{dx}=xe^x+e^x$
$=(x+1)e^x$
When $x=\ln 2$
$\frac{dy}{dx}=(1+\ln 2)e^{\ln 2}$
$=(1+\ln 2)\times 2$
$\therefore$ Gradient $=2(1+\ln 2)$</td><td><u>2 marks</u>: correct solution

<u>1 mark</u>: significant progress towards correct solution</td></tr>
<tr><td>(e) (i)</td><td>$f(x)=e^{2x}(1-x)$
<table><tr><td>x</td><td>–3</td><td>–2</td><td>–1</td><td>0</td><td>1</td></tr><tr><td>$f(x)$</td><td>0.01</td><td>0.05</td><td>0.27</td><td>1</td><td>0</td></tr></table></td><td><u>1 mark</u>: both values correct</td></tr>
<tr><td>(e) (ii)</td><td>$A=\frac{h}{2}(a+b)$ with 4 applications
$=\frac{1}{2}\left[(0.01+0.05)+(0.05+0.27)+(0.27+1)+(1+0)\right]$
$=\frac{1}{2}\times 2.65$
$=1.325\text{ units}^2$</td><td><u>2 marks</u>: correct solution

<u>1 mark</u>: significant progress towards correct solution</td></tr>
</table>

(e) (iii)	Answer is an underestimate since the first three applications are very small overestimates (since part of each trapezium sits outside the curve) and the final application is a very large underestimate as it gives the area of a triangle sitting inside the curve.	**<u>1 mark</u>:** correct explanation